Product differentiation and firm profits in a Hotelling model with endogenous timing*

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Abstract

This paper revisits the relationship between product differentiation and firm profits. We analyze a Hotelling duopoly with endogenous timing choice. There exists an equilibrium where an efficient firm becomes the leader whereas an inefficient firm follows. We show that, when the cost difference is sufficiently large, a decrease in the degree of product differentiation increases the efficient leader's profit. Moreover, the joint profit of those firms may increase with product substitutability.

Keywords: Horizontal product differentiation, Profit, Firm asymmetry, Endogenous timing, Observable delay, Competitor collaboration

JEL Classification Numbers: L13, L25, M21, M31

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1 Introduction

The relationship between competition and firm profits is one of the fundamental questions in industrial organization and marketing strategy.[1] In contrast to conventional wisdom, several recent studies raise instances where increasing competition, which is measured by the number of firms or degree of product differentiation, may actually increase firm profits.[2] Zanchettin (2006) considers Bertrand and Cournot duopoly competition with product differentiation *à la* Singh and Vives (1984), and shows that when product substitutability increases, an efficient firm can increase its profit whereas an inefficient firm unambiguously loses.[3] His result has managerial implications, namely that an efficient firm should not emphasize the distinctiveness between its product and that of a rival.

Following the standard interpretation by Cabral (2000), and Belleflamme and Peitz (2010), we employ transportation cost as the measure for product substitutability in a Hotelling model.[4] In contrast to Zanchettin (2006), it is well-known that profit decreases with product substitutability in the exogenous-location Hotelling duopoly with cost asymmetry.[5] In fact, Cabral (2000, p. 215) states that "A greater value of t corresponds to a greater degree of product differentiation. We thus conclude: The greater the degree of product differentiation, the greater the degree of market power."[6] Belleflamme and Peitz (2010, p. 51) also state that "The more products are differentiated, i.e., the higher τ, the higher the price-cost margin of the firms in equilibrium."[7] Thus, the implication of Zanchettin (2006) cannot be applied to markets with inelastic (unit) demand, such as the mobile phone market.

In contrast with the above textbook examples, we show a scenario where less differentiation

[1] Vives (2008) considers two possible measures of enhanced competitive pressure as an increase in the degree of product substitutability or in the number of competitors.

[2] See, for example, Mukherjee and Zhao (2009) and Ishida et al. (2011).

[3] Dixit (1979) is the pioneering work which examines the relationship between firm profits and product differentiation. Several papers show similar results by employing a Stackelberg model with free entry (Žigić, 2012) and a unionized duopoly (Fanti, 2013; Fanti and Meccheri, 2014). Yoshida (2016) also shows a similar result, but the driving force is quite different from that in our model.

[4] Brenner (2001) considers the transportation cost as the level of price competition.

[5] In fact, Hotelling (1929, p. 50) states that firms reach higher profits when product differentiation, measured by the transportation cost, becomes higher.

[6] t denotes the transportation cost.

[7] τ denotes the transportation cost.

can be better for the efficient firm in a Hotelling duopoly model by incorporating the observable delay game formulated by Hamilton and Slutsky (1990), which is one of the most popular and important endogenous timing games.[8] The game runs as follows. Each firm first chooses the timing of price setting, and then sets its price within the selected timing. We find that an equilibrium exists where an efficient firm becomes the leader whereas an inefficient firm becomes the follower when the cost difference is sufficiently large. In this equilibrium, the efficient leader's profit can increase with product substitutability.

The intuition behind the result is as follows. The degree of product substitutability affects the efficient leader's profit through the following three ways. First, an increase in product substitution directly reduces its price, which hurts the leader's profit. Second, an increase in product substitution makes the demand more price-elastic, which implies that the difference of the firms' prices has stronger impact on it. This benefits the firm that charges a lower price than its rival. Under some condition, since the efficient firm charges a lower price than the inefficient rival even when the efficient firm becomes leader, it can get a benefit from this effect. Third, an increase in product substitution expands the price difference between two firms, which increases the leader's market share. The combination of the latter two effects increases the efficient leader's demand significantly, while its price decreases, and consequently these positive effects can dominate the direct negative effect. That is why the efficient firm can benefits by an increase in product substitution.

The rest of this paper is organized as follows. Section 2 sets up our model and shows main results on relationships between product differentiation and firm profits. Section 3 offers some concluding remarks.

2 Model and Analysis

Consider a linear city where consumers are uniformly distributed on $[0, 1]$ with mass 1. Firm $i \in \{0, 1\}$ is located at i and is selling a homogeneous product. Let p_i denote the price of firm i and c_i denote a constant marginal cost of firm i. Assume that $c_1 > c_0 \geq 0$, and then we call firm

[8] Endogenous timing games have been explored intensively. See Pal (1998), Matsumura (1999, 2002) and Naya (2015).

0 (1) the (in)efficient firm.

Consumers buy up to one unit of product with incurring linear transportation costs. Thus, the utility of the consumer located at $x \in [0, 1]$ is given by

$$U(x) = \begin{cases} r - p_i - td_i(x) & \text{if she buys from firm } i, \\ 0 & \text{if she does not buy,} \end{cases} \tag{1}$$

where r is a gross utility, which is sufficiently large and $d_i(x)$ denotes the distance between consumer x and firm i. Following the standard interpretation by Cabral (2000), and Belleflamme and Peitz (2010, p. 115), t is the measure for product substitutability between two products.

Let $\hat{x}(p_0, p_1)$ be the consumer who is indifferent between buying from firm 0 and firm 1. The value of $\hat{x}$ is given as follows:

$$r - p_0 - t\hat{x} = r - p_1 - t(1 - \hat{x}) \Leftrightarrow \hat{x} = \frac{t + p_1 - p_0}{2t}. \tag{2}$$

Thus, the demands and the profits of firm i are

$$D_i = \min\left\{\max\left\{\frac{t + p_j - p_i}{2t}, 0\right\}, 1\right\}, \quad i \neq j, \tag{3}$$

and

$$\pi_i = (p_i - c_i)D_i, \quad i = 0, 1. \tag{4}$$

The game proceeds as follows. The firms first choose to become leader or follower, and then set their prices. If both firms become leader or both become follower, the price competition becomes a simultaneous-move game.

Hereafter, we assume that $c_1 - c_0 \leq 5t$, because when $c_1 - c_0 > 5t$, the inefficient firm is always inactive.

2.1 Outcomes in the second stage

Since the model is standard enough, we only provide the outcomes of the analysis.

2.1.1 When the firms move simultaneously

In this case, the equilibrium outcome is that of a standard Hotelling model. The equilibrium prices and profits are as follows:

$$(p_1^S, p_0^S) = \begin{cases} \left(\dfrac{3t + 2c_1 + c_0}{3}, \dfrac{3t + 2c_0 + c_1}{3}\right) & \text{if } c_1 - c_0 \leq 3t, \\ (c_1, c_1 - t) & \text{if } c_1 - c_0 > 3t, \end{cases} \tag{5}$$

$$\pi_1^S = \begin{cases} \dfrac{(3t - c_1 + c_0)^2}{18t} & \text{if } c_1 - c_0 \leq 3t, \\ 0 & \text{if } c_1 - c_0 > 3t. \end{cases} \tag{6}$$

$$\pi_0^S = \begin{cases} \dfrac{(3t - c_0 + c_1)^2}{18t} & \text{if } c_1 - c_0 \leq 3t, \\ c_1 - t - c_0 & \text{if } c_1 - c_0 > 3t, \end{cases} \tag{7}$$

Note that if $c_1 - c_0 > 3t$, the equilibrium outcome becomes the corner solution that the efficient firm can get all consumers by limit pricing.

2.1.2 When the efficient firm is leader

Since we assume $c_1 - c_0 \leq 5t$, a corner solution does not emerge in this case. Solving the maximization problem, we have the equilibrium prices and profits as follows:

$$p_1^F = \frac{5t + 3c_1 + c_0}{4}, \quad p_0^L = \frac{3t + c_0 + c_1}{2}, \tag{8}$$

and

$$\pi_1^F = \frac{(5t - c_1 + c_0)^2}{32t}, \quad \pi_0^L = \frac{(3t - c_0 + c_1)^2}{16t}. \tag{9}$$

We get the following lemma.

Lemma 1 *The equilibrium demand of the efficient firm, $D_0^L = (3t - c_0 + c_1)/8t$, decreases with t.*

Proof. Differentiating the efficient firm's demand with respect to t, we have

$$\frac{dD_0^L}{dt} = -\frac{c_1 - c_0}{8t^2} < 0. \tag{10}$$

■

The intuition behind the result is as follows. The equilibrium price-difference $(p_0^L - p_1^F)$ monotonically increases with t. This is because when t is relatively high, the efficient leader sets a sufficiently high price in anticipation of the follower's undercutting. However, smaller t no longer leaves much room for undercutting. Note that the sequential move is important for this effect, rather than cost differentials. On the other hand, when t becomes small, the demand becomes price sensitive, which benefits the firm whose price is lower than that of the rival. Unless the marginal costs are symmetric, the first effect could be dominant.

Figure 1 shows a numerical example. Note that when the efficient firm's price becomes lower than that of the inefficient one for low t (i.e., firm 0's demand is more than one half), the marginal increment of the firm 1's demand significantly increases when t is further reduced. The critical cutoff value of t, in which the efficient leader's equilibrium price starts to exceed the inefficient follower's price, is $c_1 - c_0$.

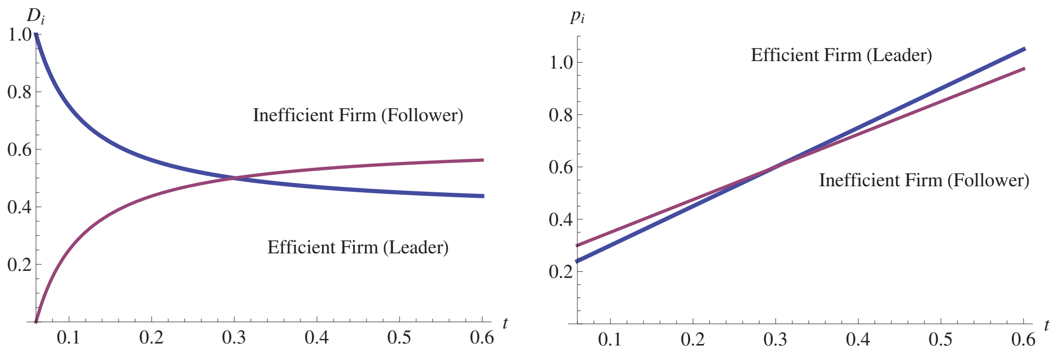

Figure 1: The relationships between the demands (left) and the prices (right) and product substitutability. ($c_1 = 0.3, c_0 = 0.$)

2.1.3 When the inefficient firm is leader

The analysis is parallel to the previous case. Therefore, the interior solutions can simply be obtained by swapping indices in the previous case. However, even when $c_1 - c_0 \leq 5t$, a corner solution emerges in this case. The equilibrium prices and profits are as follows:[9]

$$(p_1^L, p_0^F) = \begin{cases} \left(\dfrac{3t + c_1 + c_0}{2}, \dfrac{5t + 3c_0 + c_1}{4}\right) & \text{if } c_1 - c_0 \leq 3t, \\ (c_1, c_1 - t) & \text{if } c_1 - c_0 > 3t, \end{cases} \tag{11}$$

$$\pi_1^L = \begin{cases} \dfrac{(3t - c_1 + c_0)^2}{16t} & \text{if } c_1 - c_0 \leq 3t, \\ 0 & \text{if } c_1 - c_0 > 3t. \end{cases} \tag{12}$$

$$\pi_0^F = \begin{cases} \dfrac{(5t - c_0 + c_1)^2}{32t} & \text{if } c_1 - c_0 \leq 3t, \\ c_1 - t - c_0 & \text{if } c_1 - c_0 > 3t, \end{cases} \tag{13}$$

2.2 Timing choice in the first stage

Here, we consider the timing choice. The endogenous timing problem can be summarized in Tables 1 and 2.

Firm 0\1	Leader	Follower
L	$\dfrac{(3t - c_0 + c_1)^2}{18t}, \dfrac{(3t + c_0 - c_1)^2}{18t}$	$\dfrac{(3t - c_0 + c_1)^2}{16t}, \dfrac{(5t - c_1 + c_0)^2}{32t}$
F	$\dfrac{(5t - c_0 + c_1)^2}{32t}, \dfrac{(3t - c_1 + c_0)^2}{16t}$	$\dfrac{(3t - c_0 + c_1)^2}{18t}, \dfrac{(3t + c_0 - c_1)^2}{18t}$

Table 1: When $c_1 - c_0 \leq 3t$.

Firm 0\1	Leader	Follower
L	$c_1 - t - c_0, \quad 0$	$\dfrac{(3t - c_0 + c_1)^2}{16t}, \dfrac{(5t - c_1 + c_0)^2}{32t}$
F	$c_1 - t - c_0, \quad 0$	$c_1 - t - c_0, \quad 0$

Table 2: When $c_1 - c_0 > 3t$.

The payoff matrices lead to the following lemma.

[9]Since there exist continuously many equilibrium outcomes, We pick up the outcome where firm 1 chooses marginal cost pricing. In fact, any p_1 above c_1 can be a part of subgame perfect equilibrium. Unless firm 1 sets an extremely high price, the analysis in the successive subsection does not change.

Lemma 2 *There exist multiple equilibria where the efficient firm becomes leader and the inefficient firm becomes leader regardless of the inefficient firm's marginal cost.*

This result directly follows the result of Hamilton and Slutsky (1990). They show that, in the observable delay games, the asymmetric timing outcome occurs with strategic complementarity.[10]

We wish to say that the equilibrium outcome where the inefficient firm becomes leader is not plausible when $c_1 - c_0 > 3t$, because firm 1 employs a weakly dominated strategy in that case. Taking this matter into account, we obtain the following proposition.

Proposition 1 *Consider a trembling hand perfect equilibrium of the reduced game in the first stage. When $c_1 - c_0 > 3t$, there exists a unique equilibrium outcome where only the efficient firm becomes leader.*

Proof. Osborne and Rubinstein (1994, Proposition 248.2) state that a strategy profile in a finite two-player strategic game is a trembling hand perfect equilibrium if and only if it is a mixed strategy Nash equilibrium, and the strategy of neither player is weakly dominated.[11] Since playing L for firm 1 is weakly dominated by F, the strategy profile (F, L) cannot be a trembling hand perfect equilibrium.[12] ■

Henceforth, we investigate the equilibrium type where the efficient firm becomes leader. That is, we focus on the case when $c_1 - c_0 > 3t$. We are interested in how the degree of product substitutability affects the firms' profits.

Proposition 2 *When $3t < c_1 - c_0 < 5t$, the efficient firm's profit increases with the degree of product substitutability. Formally, $d\pi_0^L/dt < 0$.*

[10] In contrast, Pan (2018) shows that the asymmetric timing outcome occurs with strategic substitutability when a demand function is kinked or cubic type, which makes the resulting profit functions are globally nonconcave.

[11] Note that a mixed strategy includes a pure strategy.

[12] This type of equilibrium is payoff dominant. Comparing two equilibria, (L, F) and (F, L), we have

$$\frac{(3t - c_0 + c_1)^2}{16t} - (c_1 - t - c_0) > 0,$$

which implies that the efficient firm prefers the outcome of (L, F). Since the inefficient firm prefers (L, F) outcome, (L, F) dominates the other by using the payoff dominance criterion.

Proof. We now show that $d\pi_0^L/dt < 0$. Differentiating the efficient firm's profit with respect to t, we have

$$\frac{d\pi_0^L}{dt} = \frac{(3t - c_0 + c_1)(3t + c_0 - c_1)}{16t^2} < 0 \Leftrightarrow c_1 - c_0 > 3t. \tag{14}$$

Note that the condition satisfies the condition for the interior solution (i.e., $c_1 - c_0 < 5t$). ■

The degree of product substitutability, t, affects the efficient leader's profit through the following three ways. First, a decrease in t directly reduces its price, which harms the leader's profit. Second, a decrease in t results in more price-elastic demand, which implies that the difference of the firms' prices gets stronger impact on it. This can benefit the firm that charges a lower price than its rival. Under the condition of the proposition, since firm 0 charges a lower price than firm 1, firm 0 can get a benefit from this effect. Third, a decrease in t expands the price difference between two firms, which increases the leader's market share. The latter two effects increase the efficient leader's demand significantly (by Lemma 1), while its price decreases, and consequently these positive effects can dominate the direct negative effect. Figure 2 shows a numerical example.

Note that the second effect is similar to the *selection effect*, pointed out by Zanchettin (2006). In order to understand the reason why such a result does not hold in the simultaneous-move case, it is more important to understand the third effect. In that case (i.e., the standard Hotelling model), the third effect does not appear because the price difference between the firms does not depend on product substitutability.

Note further that, when $c_1 - c_0 > 3t$, it is in equilibria with limit pricing that the efficient firm's profit increases with the degree of product substitutability. However, the logic behind the result is completely different with Proposition 2. In this case, since the efficient firm can get whole demand by limit pricing, an decrease in t reduces the transportation cost which the efficient firm must incur.

We next show that this result may apply to business strategies such as joint activity among competitors. We get the following result.

Proposition 3 *An increase in the degree of product substitutability may increase the joint profit of the firms. Formally,* $d(\pi_0^L + \pi_1^F)/dt < 0$.

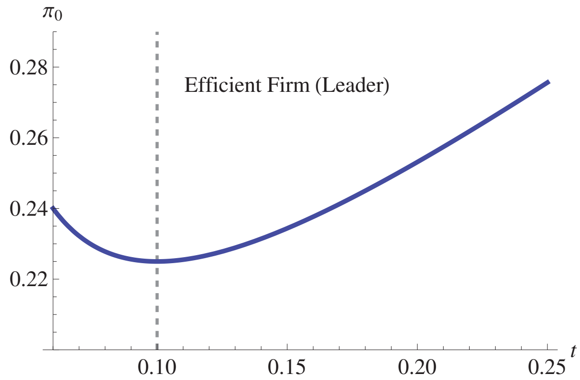

Figure 2: Relationship between firm 0's profit and product substitutability. ($c_1 = 0.3$, $c_0 = 0$.)
Note: The dotted line denotes $(c_1 - c_0)/3t$.

Proof. We show that the sum of those two firms decreases with the transportation cost. Differentiating the joint profit of the firms with respect to t, we have

$$\frac{d(\pi_0^L + \pi_1^F)}{dt} = \frac{43t^2 - 3(c_1 - c_o)^2}{32t^2} < 0, \Leftrightarrow c_1 - c_0 > \sqrt{\frac{43}{3}}t \approx 3.786t.$$

Note that this value is lower than $5t$, which assures the condition for the proposition. ■

This result states that the benefit from a reduction of product differentiation for the efficient leader can be large enough to compensate the inefficient rival's loss. Cost asymmetry between the firms plays an important role for this result. Proposition 3 implies that those firms may have an incentive to jointly make their products less differentiated, through monetary transfers. For example, such business strategy may be possible through a joint research/production venture.[13]

3 Concluding Remarks

We investigate the possibility of a "reversal result" in a Hotelling model. That is, we examine the following question: does an increase in the product substitution always harm firm profits? To this end, we analyze a Hotelling duopoly with endogenous timing choice. It is found that an increase in the degree of product substitutability can increase the profit of the efficient leader. This is in stark contrast to Hotelling's (1929) argument. Moreover, that can increase the joint

[13]Ghosh and Morita (2012) study possible trade-offs between a reduction of product distinctiveness and fixed-cost savings associated with competitor collaborations. In contrast to ours, they analyze the collaborations reducing the distinctiveness between partners' products but increasing that between their products and an outsider's.

profit as well.

The intuition behind the result is as follows. An increase in product substitution makes the demand more price-elastic, which implies that the difference of the firms' prices has stronger impact on it. This benefits the firm that charges a lower price than its rival. Under such a condition, since the efficient firm charges a lower price than the inefficient rival even when the efficient firm becomes leader, it can get a benefit from this effect. Additionally, an increase in product substitution expands the price difference between two firms, which increases the leader's market share. The combination of two effects expands the efficient leader's demand, although its price decreases. That is why the efficient firm can benefits by an increase in product substitution.

To the best of our knowledge, no previous paper has analyzed the relationship between product differentiation and profit in a Hotelling model with endogenous timing choice. Both of our results might provide a new insight into firms' business strategies (e.g., advertising or joint venture). For example, our first result implies that an efficient leader should not emphasize the distinctiveness between its product and that of a follower.

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