The Optimal Trade-off between Aggregate Abatement Cost and Monitoring and Enforcement Cost

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Abstract: This paper studies the regulator who minimizes the sum of the aggregate abatement cost and the monitoring and enforcement (ME) cost to attain a given target aggregate emission reduction via taxation, and analyzes the optimal tax rate and ME level according to the target. We find that, except for the case where the target is very low, it is too costly to raise the ME level until the aggregate abatement cost is minimized. Some trade-offs between the aggregate abatement cost and the ME cost are desirable. Thus, the efficient allocation of emission reductions holds only within the coalition of relatively compliant firms. Seeking a more ambitious target, the regulator should lay greater stress on taxation than on ME and make more of a compromise on the abatement cost efficiency in order to offset the rise in the ME cost. Hence, the size of the efficient coalition becomes smaller. An extremely high target may preclude the opportunity to trade off the ME cost against the aggregate abatement cost because of the high marginal ME cost. Then, the regulator should abandon the idea of the abatement cost efficiency and consider the target attainability alone.

Keywords: abatement cost, compliance, efficiency, environmental taxation, monitoring and enforcement

JEL classification: D62, H21, H23, H26

1 Introduction

Use of environmental taxation is now widespread especially among the EU countries attempting to regulate greenhouse gas emissions.[*1] Proponents of the taxation often insist that uniform unit taxation on emissions has a crucial advantage over fixed quotas: it minimizes the aggregate abatement cost of attaining any predetermined target(s) of aggregate emission reduction. This abatement cost efficiency (cost-effectiveness) of taxation was formally established by Baumol and Oates (1971).[*2]

However, abatement cost efficiency and, in the extreme, target attainability of environmental

[*1] OECD (2001) provides a comprehensive discussion on environmentally related taxes in OECD members. See OECD/EEA (2010) for more recent information.

[*2] The exposition of the abatement cost efficiency includes Baumol and Oates (1988), Hanley et al. (2007), and Muller and Mendelsohn (2009) among others.

taxation become doubtful when we consider the imperfect tax compliance of regulated polluters. Although Sandmo (2002) argued that the unit tax system is able to attain the target at minimum cost even if polluters attempt tax evasion, Shiota (2008) has demonstrated that Sandmo's proposition holds only if the targets are low enough or the polluters are sufficiently risk averse.

When seeking relatively large target reductions via taxation, the regulator must impose a substantial tax on emissions, which is a strong incentive for the polluters to evade taxes. If some polluters take the risk of 100% tax evasion because of high tax rates, then the marginal cost(s) of emission reduction (MCER) is no longer equalized across all the polluters and the resulting allocation of emission reductions fails to be efficient. Additionally, if all the polluters commit 100% tax evasion, then any further emission reductions are impossible whatever high tax rates the regulator levies.*3

In order to recover the efficiency (and attainability) of the unit tax system, it is essential for the regulator to ensure tax compliance even when rates are high. We may advise, then, that the regulator reinforce monitoring and enforcement (ME) activities against tax evasion. However, such a policy itself will require some extra social expenditure, which may exceed savings in abatement costs. It is necessary, therefore, to consider in what ways the regulator should trade off aggregate abatement cost against ME cost when we study how to minimize the total social cost of realizing a given aggregate emission reduction.*4*5

Let me now briefly explain the model and conclusions of the present paper. We adopt a variant of Sandmo's (2002) model to formulate the behavior of individual polluting firms and introduce the regulator who seeks to minimize the social cost of attaining the target emission reduction.

Each firm is required to report emissions and pay taxes based on those reports. The regulator implements ME and imposes penalties on the firms which are caught cheating. Every firm has its own subjective evaluation of expected marginal penalty (SEEMP), which depends on the ME level. Weighing the tax rate against the ME level, each firm decides on the actual emission level and the percentage of emissions to report that will maximize expected profits. Firms will evade 0% of taxes and set the actual emissions where MCER is equal to the tax rate if the tax rate is lower than SEEMP. They will evade

*3 See Shiota (2008) for further discussion of these points.

*4 Malik (1992) showed that conditions which minimize the aggregate abatement cost are different from those which minimize the ME cost and argued that environmental taxation may become more costly than fixed quotas once both these costs are considered. By investigating the optimal environmental tax rate and ME level, the present paper reveals the conditions to minimize the sum of these costs and how they are influenced by the target level. Although our results also have important implications for the choice of the environmental policy instruments, our focus is different from his.

*5 The dual problem of Macho-Stadler and Pérez-Castrillo (2006) becomes the one, which is similar to ours, where the regulator designs the ME schemes so as to minimize the ME cost while achieving a given aggregate emission reduction. However, in their setting the aggregate abatement cost and choice of the tax rate are ignored. Instead, they assume that the regulator has complete information about the characteristics of each firm and detail the way to customize ME level according to the type of firms. They conclude that all the firms will commit 100% tax evasion at the optimum. As we shall mention further on, our result coincides with theirs when the target emission reductions are extremely large.

100% and set the emissions where MCER is equal to SEEMP if the rate is higher.

If the regulator raises the tax rate, the aggregate emission reduction will increase unless all the firms commit 100% tax evasion. However, the abatement cost efficiency will decline, because a rise in the tax rate gives a stronger incentive to evade taxes and thus the proportion of firms whose MCER are equalized falls.

If the regulator strengthens ME, SEEMP of each firm will increase. As a result, the aggregate emission reduction will increase when there exists a firm which commits 100% tax evasion. Additionally, the abatement cost efficiency will improve, because the proportion of firms whose MCER are equalized expands. However, an increase in ME will come at some extra costs to the society.

Based on the foregoing, we study the optimal tax rate and ME level according to the target emission reductions. We will find that the target level has a decisive influence on the properties of optimal environmental policy.

If the target is set very low, it can be attained just by imposing a low tax rate. Any ME activity is simply wasteful, because all firms will readily comply with the taxation. Thus, MCER are equalized for all the firms without expensive ME. Consequently, the aggregate abatement cost to the overall economy is minimized and the ME cost is zero at the optimum. There is no resulting trade-off between the aggregate abatement cost and the ME cost.

When the target is raised somewhat higher, the tax rate becomes substantial and firms have an incentive to evade taxes. In such cases, the regulator had better allow the least compliant firms to commit 100% tax evasion. It becomes too expensive to raise the ME level high enough that no firms will totally evade the taxes. Some trade-offs between the aggregate abatement cost and the ME cost are necessary, and efficient allocation of emission reductions holds only within the coalition of relatively compliant firms at the optimum.

A more ambitious target requires both a higher tax rate and a stricter ME. In formulating policy, the regulator should lay greater stress on taxation than on ME and be prepared to compromise on the aggregate abatement cost in order to offset the rise in ME cost. However, the consequence of such a policy is that the size of the efficient coalition becomes smaller.

In the most extreme case, the target is set so high the regulator should not trade off any ME cost against the aggregate abatement cost because of the high marginal ME cost. The regulator is forced to abandon the idea of abatement cost efficiency and seek to minimize only the ME cost. Under these circumstances, all the firms attempt 100% tax evasion and the size of the efficient coalition is negligible.

The remainder of this paper proceeds as follows: Section 2 analyzes the behavior of polluting firms facing environmental regulation. Section 3 studies the regulator who minimizes the sum of the aggregate abatement cost and the ME cost to attain a given target. Optimal environmental taxation and ME are characterized. Section 4 summarizes the implications for environmental policy of realizing a predetermined reduction. The Appendix contains proofs of the propositions.

2 Individual Reaction toward Environmental Policy

This section studies how each firm responds to environmental taxation and ME activities, and what losses it will suffer.

Consider an economy consisting of a continuum of firms, where each firm is represented by a point $j = (j_1, j_2) \in [0,1] \times [0,1]$. Firm j produces a single output x_j and sells it at a competitive price p. In the production process, firm j emits a homogeneous pollutant e_j. Firms will incur some extra costs if they attempt to reduce emissions.

The technology of firm j is represented by an additively separable cost function C_j:

$$C_j(x_j, e_j) = C_j^X(x_j) + C_j^E(e_j),$$

$$\frac{\partial C_j^X}{\partial x_j} > 0, \ \frac{\partial^2 C_j^X}{\partial x_j^2} > 0, \tag{1}$$

$$\lim_{x_j \downarrow 0} \frac{\partial C_j^X(x_j)}{\partial x_j} = 0, \tag{2}$$

$$\lim_{x_j \to +\infty} \frac{\partial C_j^X(x_j)}{\partial x_j} = +\infty, \tag{3}$$

For analytical clarity, we make the following additional assumption about technology:

Assumption 1　We assume that C_j^E is quadratic in e_j:

$$C_j^E(e_j) = \frac{k}{2}e_j^2 - ke^0 e_j + \hat{C}, \tag{4}$$

where

$$e^0 = (\bar{e}^0 - \underline{e}^0) j_1 + \underline{e}^0,$$
$$\bar{e}^0 > \underline{e}^0 > 0,$$
$$k > 0,$$
$$\hat{C} \geq 0.$$

Since $j_1 \in [0,1]$, the maximal emission level e^0 is uniformly distributed on the interval $[\underline{e}^0, \bar{e}^0]$ and the marginal cost of emission reduction (MCER)[*6] when $e_j = e$:

$$-\frac{\partial C_j^E}{\partial e_j}(e) = k(e^0 - e)$$

is uniformly distributed on the interval $[k(\underline{e}^0 - e), k(\bar{e}^0 - e)]$.[*7]

There is a regulator who implements an environmental policy (t, s) to control emissions, where $t \geq 0$ is the rate of uniform unit taxation on emissions and $s \geq 0$ is the degree of ME to ensure tax compliance. All the firms are required to report their emissions and pay taxes according to the reported level. Letting $b_j \in [0,1]$ denote the truth-telling rate of firm j, we can write the taxes that the firm j pays as tb_je_j.

The regulator notifies firms that it will conduct surprise inspections and impose penalties on any which are found to be evading taxes. Firm j perceives that the probability of conviction is $\alpha_j \in (0,1]$, and that the penalty $\rho_j[\cdot]$ is a function of the level of underreported emissions $(1-b_j)e_j$.

We can interpret the product $\alpha_j \rho_j[(1-b_j)e_j]$ as the subjective evaluation of the expected penalty by firm j. Let us make the following assump-

[*6] Cost of emission reduction means the increase in production cost resulting from the reduction in emissions.

[*7] Without loss of generality, we postulate that a firm with a larger j_1 has a higher MCER when emission levels are the same.

tion about the subjective perception of penalty for tractability:

Assumption 2 We assume that the subjective evaluation of the expected penalty is linear in the level of underreporting:

$$\alpha_j \rho_j[(1-b_j)e_j] = \gamma(1-b_j)e_j$$

where

$$\gamma = (\bar{a} - \underline{a})j_2 + \underline{a} + s,$$
$$\bar{a} > \underline{a} > 0.$$

The coefficient which represents the subjective evaluation of expected marginal penalty (SEEMP), γ, depends on the ME level[*8] and is uniformly distributed on the interval $[\underline{a} + s,\ \bar{a} + s]$.[*9] We shall suppose that although the regulator is unable to identify the SEEMP of each individual firm, the distribution of SEEMP among all firms is known.

The net profit of firm j becomes π_j^d if it is detected underreporting, and π_j^u if not:

$$\pi_j^d = px_j - C_j^X(x_j) - C_j^E(e_j) - tb_je_j - \rho_j[(1-b_j)e_j], \quad (5)$$
$$\pi_j^u = px_j - C_j^X(x_j) - C_j^E(e_j) - tb_je_j. \quad (6)$$

We suppose that all the firms are risk neutral. Since the net expected profit is given by $\alpha_j\pi_j^d + (1-\alpha_j)\pi_j^u$, firm j solves

$$\begin{aligned} \underset{x_j,e_j,b_j}{\text{Max}} \quad & px_j - C_j^X(x_j) - \frac{k}{2}e_j^2 + ke^0e_j \\ & -\hat{C} - tb_je_j - \gamma(1-b_j)e_j \\ \text{s.t.} \quad & x_j \geq 0, \\ & e_j \geq 0, \\ & b_j \in [0,1]. \end{aligned}$$

After some conventional calculations, we can clarify the behavior of firms. Because of our interests, we suppose that $\underline{e}^0$ is sufficiently large and focus only on the case where $e_j > 0$ for all $j \in [0,1] \times [0,1]$.

We begin with the output decision:

Proposition 1 Suppose $p > 0$. Then the output level of firm j, $x_j(p)$, satisfies

$$x_j(p) > 0, \quad x_j'(p) > 0.$$

Since firms have additively separable technologies, output level is independent of environmental policy (t, s).

The following two propositions characterize the properties of the truth-telling rate and emission level. For the same reason as in the case of the output level, their properties are independent of the output price.

Proposition 2 If the regulator implements an environmental policy (t, s), then the truth-telling

[*8] If the regulator commits itself to increasing its monitoring activities, such as inspections by the regulatory agencies, subjective probability of conviction of firm j will increase: α_j will rise. If the regulator commits itself to strengthening enforcement mechanisms, such as penalty schemes, the evaluation of penalty by firm j will rise. Such a policy can be represented by an upward shift in $\rho_j[\cdot]$. However, we treat monitoring and enforcement as a set here for the sake of simplicity.

[*9] Without loss of generality, we postulate that a firm with a larger j_2 has a higher SEEMP.

rate of firm j, $b_j(t, s)$, is given by

$$b_j(t,s) = \begin{cases} 1 & \text{if } t \in [0, \gamma), \\ [0,1] & \text{if } t = \gamma, \\ 0 & \text{if } t > \gamma. \end{cases} \tag{7}$$

Suppose that (t, s) is given and j_2' satisfies

$$j_2' = \frac{t - s - \underline{a}}{\bar{a} - \underline{a}}. \tag{8}$$

Then, Proposition 2 shows: (i) firm $j \in [0, 1] \times (j_2', 1]$ is perfectly compliant; (ii) firm $j \in [0, 1] \times \{j_2'\}$ is indifferent to the proportion of taxes it pays; and (iii) firm $j \in [0, 1] \times [0, j_2')$ commits 100% tax evasion.

Proposition 3 If the regulator implements an environmental policy (t, s), then the emissions decision made by firm j, $e_j(t, s)$, is given by

$$e_j(t,s) = \begin{cases} e^0 - \frac{t}{k} & \text{if } t \in [0, \gamma), \\ e^0 - \frac{t}{k} = e^0 - \frac{\gamma}{k} & \text{if } t = \gamma, \\ e^0 - \frac{\gamma}{k} & \text{if } t > \gamma. \end{cases} \tag{9}$$

Let j_2' be the same as in (8). From Assumption 1, MCER at e_j is $k(e^0 - e_j)$. Therefore, Proposition 3 shows: (i) firm $j \in [0, 1] \times (j_2', 1]$ determines the emission level so that MCER may equal the tax rate; (ii) firm $j \in [0, 1] \times \{j_2'\}$ determines so that MCER may equal the tax rate as well as SEEMP; and (iii) firm $j \in [0, 1] \times [0, j_2')$ determines so that MCER may equal SEEMP which is smaller than the tax rate.

We have understood how each individual firm responds to the environmental tax rate and ME activities. Closing the section, we investigate what losses each firm will suffer when the environmental policy is implemented.

Because we suppose that the objective of each firm is its net expected profit, it is relevant to consider the loss in net expected profit to be the loss of each firm by the environmental policy. From (5) and (6), we understand that the loss in net expected profit consists of the sum of the loss in gross profit, tax payments, and the expected penalties. The loss in gross profit consists of the sum of the reduction in sales, the increase in production cost caused by change in output, and the increase in production cost brought about by the reduction in emissions. We shall refer to the loss in the gross profit of firm j as the abatement cost of firm j.

In our model, however, every firm has an additive separable technology. Thus, production decisions and market price are independent of environmental policy. Consequently, the reduction in sales is zero, and the increase in production cost caused by change in output is also zero. Therefore, the abatement cost is equal to the increase in production cost brought about by the reduction in emissions.

Finally, the loss in net expected profit of firm j resulting from the environmental policy (t, s) is given by

$$\begin{aligned} C_j^E(e_j(t,s)) - C_j^E(e_j(0,0)) + t b_j(t,s) e_j(t,s) \\ + \gamma(1 - b_j(t,s)) e_j(t,s). \end{aligned} \tag{10}$$

We shall now move on to consider the least expensive way for the economy to realize a given target under incomplete tax compliance.

3 The Regulator's Problem

In this section, we analyze the regulator who minimizes the total cost to the economy of attaining a predetermined target emission reduction.

After reconsidering the cost the economy will bear when it attempts to realize a given target, we characterize the optimal tax rate and the optimal ME level according to the targets. We will find that the regulator should trade off the aggregate abatement cost for the ME cost except in cases where the target is very low.

Suppose the target level of aggregate emission reduction $\bar{R} > 0$ is determined exogenously. In order to attain the target, the regulator imposes a tax on emissions at the rate $t \geq 0$ and implements ME at the level $s \geq 0$ to ensure tax compliance.*10

We now aggregate the individual response to environmental policy (t, s) examined in the previous section. Since each firm is represented by the point $j = (j_1, j_2) \in [0,1] \times [0,1]$ and the emission decision satisfies Proposition 3, the aggregate emission reduction is given by

$$\int_0^1 \int_0^1 [e_j(0,0) - e_j(t,s)]\, dj_1\, dj_2. \quad (11)$$

The following proposition shows a tractable representation of (11).

Proposition 4 Suppose the regulator implements an environmental policy (t, s). Then, the aggregate emission reduction has the following properties:

$$\int_0^1 \int_0^1 [e_j(0,0) - e_j(t,s)]\, dj_1\, dj_2$$
$$= \begin{cases} \frac{t}{k} & \text{if } 0 \leq t \leq \underline{a} + s, \\ \frac{1}{2k(\bar{a}-\underline{a})}[2t(\bar{a}+s) - t^2 - (\underline{a}+s)^2] & \\ \qquad \text{if } \underline{a} + s \leq t \leq \bar{a} + s, & \\ \frac{\bar{a}+\underline{a}+2s}{2k} & \text{if } t \geq \bar{a} + s. \end{cases}$$

Next we consider two costs the economy must bear when the policy is carried out. The first is losses incurred by polluting firms which are forced to cut down on their emissions. (10) indicates that the loss of each firm is the sum of its abatement cost, tax payments, and expected penalties. Since taxes and penalties are simply transfer payments when the economy is viewed as a whole, we can exclude them and focus solely on the abatement costs. For the same reason given in our discussion of aggregate emission reduction, the aggregate abatement cost is given by

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2. \quad (12)$$

The following proposition shows a tractable representation of (12).

Proposition 5 Suppose the regulator implements an environmental policy (t, s). Then, the aggregate abatement cost has the following properties:

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2$$
$$= \begin{cases} \frac{t^2}{2k} & \text{if } 0 \leq t \leq \underline{a} + s, \\ \frac{1}{6k(\bar{a}-\underline{a})}[3t^2(\bar{a}+s) - 2t^3 - (\underline{a}+s)^3] & \\ \qquad \text{if } \underline{a} + s \leq t \leq \bar{a} + s, & \\ \frac{1}{6k}[\bar{a}^2 + \bar{a}\underline{a} + \underline{a}^2 + 3s(\bar{a}+\underline{a}) + 3s^2] & \\ \qquad \text{if } t \geq \bar{a} + s. & \end{cases}$$

We move on to the second cost the regulator must pay when it implements ME. The previous section shows that each firm will not respond to any rise in the tax rate, if the tax rate exceeds its SEEMP. Therefore, to prompt firms to further reduce emissions, the regulator needs to strengthen ME so that SEEMP will rise. However, since ME

*10 Without loss of generality, we set the minimum level of ME at 0 for normalization.

activities require some resources, it will cause extra costs to the economy. Let $F(s)$ be the cost of ME where

$$F'(s) > 0,\ \forall\ s > 0;\ F'(0) = 0;\ F'' > 0. \quad (13)$$

Based on (12) and (13), we define the total cost of the environmental policy as follows:

Definition 1　The total cost that the economy will incur when the regulator implements an environmental policy (t, s) is the sum of the aggregate abatement cost and ME cost:

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2 + F(s).$$

We now take up the problem of the regulator who chooses $t \geq 0$ and $s \geq 0$ to minimize the total cost while attaining $\bar{R} > 0$. Before proceeding, we establish the following two lemmas to simplify our reasoning. Lemma 1 below enables us to focus only on the cases where $t \geq \underline{a} + s$ or $s = 0$:

Lemma 1　Suppose there is an environmental policy (t', s') such that $t' < \underline{a} + s'$ and $s' > 0$. Then it is not optimal.

Simply stated, Lemma 1 means that if the tax rate is low enough to ensure compliance by all the firms, the regulator should not implement any costly ME activities.

Next, making use of Lemma 2 below, we need only examine the case where $t = \bar{a} + s$ as the representative of all the cases where $t \geq \bar{a} + s$.

Lemma 2　An environmental policy (t^*, s^*) which satisfies $t^* = \bar{a} + s^*$ is optimal if and only if every policy (t^{**}, s^*) which satisfies $t^{**} > \bar{a} + s^*$ is optimal.

In other words, if the tax rate is high enough to make all the firms commit 100% tax evasion at a certain ME level, then all the policies will produce the same results.

We move on to characterize the optimal environmental policy according to $\bar{R}$. We begin with the cases where the targets are sufficiently low.

Proposition 6　Suppose $\bar{R} \leq \underline{a}/k$, then:

(i) the optimal environmental policy is $(k\bar{R},\ 0)$.

(ii) MCER of each firm j is equal to the tax rate $k\bar{R}$ for all $j \in [0,1] \times [0,1]$ when $(k\bar{R},\ 0)$ is implemented.

When the target is very low, the MCER of each firm is negligible and thus none will have an incentive to evade taxes. Therefore, no additional ME activities are necessary to ensure perfect tax compliance and any costly ME would simply be wasteful.

The aggregate abatement cost to the overall economy is minimized and ME cost is zero at the optimum. Hence, there is no trade-off between the aggregate abatement cost and ME cost.

We move on to the cases where $\bar{R} > \underline{a}/k$. From Proposition 4, we know that any environmental policy (t, s) which satisfies $t < \underline{a}$ and $s = 0$ can not reduce emissions more than $\underline{a}/k$. Thus, such policies will not be the optimal policy when $\bar{R} > \underline{a}/k$. Considering Lemma 1 and Lemma 2 additionally, we understand that we have only to study the following reduced regulator's problem (RRP) :

$$\min_{t,s} \frac{1}{6k(\bar{a}-\underline{a})}[3t^2(\bar{a}+s)-2t^3-(\underline{a}+s)^3]+F(s)$$

$$\text{s.t.} \frac{1}{2k(\bar{a}-\underline{a})}[2t(\bar{a}+s)-t^2-(\underline{a}+s)^2] \geq \bar{R}, \tag{14}$$

$$s \geq 0, \tag{15}$$

$$t \geq \underline{a}+s, \tag{16}$$

$$t \leq \bar{a}+s. \tag{17}$$

In order to characterize the optimal environmental policy when $\bar{R} > \underline{a}/k$, we make use of the following upper critical level $\bar{R}^{\#}$.*11

Definition 2 The upper critical level $\bar{R}^{\#}$ is the level of the aggregate emission reduction that satisfies

$$F'(k\bar{R}^{\#}-\frac{\bar{a}+\underline{a}}{2})=\frac{\bar{a}-\underline{a}}{2k}.$$

From (13), it is clear

$$\bar{R} < \bar{R}^{\#} \Leftrightarrow F'(k\bar{R}-\frac{\bar{a}+\underline{a}}{2}) < \frac{\bar{a}-\underline{a}}{2k}, \tag{18}$$

$$\bar{R} > \bar{R}^{\#} \Leftrightarrow F'(k\bar{R}-\frac{\bar{a}+\underline{a}}{2}) > \frac{\bar{a}-\underline{a}}{2k}. \tag{19}$$

The following Proposition 7 is concerned with the case where the target level is intermediate.

Proposition 7 Suppose $\bar{R} \in (\underline{a}/k, \bar{R}^{\#})$, then:

(i) the optimal environmental policy (t^*, s^*) satisfies

$$t^* \in (\underline{a}+s^*, \bar{a}+s^*), \ s^* > 0.$$

(ii) MCER of each firm j is equal to t^* for all $j \in [0,1] \times [(t^*-\underline{a}-s^*)/(\bar{a}-\underline{a}), 1]$, and is equal to its SEEMP and smaller than t^* for all $j \in [0,1] \times [0, (t^*-\underline{a}-s^*)/(\bar{a}-\underline{a})]$ when (t^*, s^*) is implemented.

(iii) the regulator should raise not only the tax rate but also the ME level to attain a higher target, although the rise in the tax rate should exceed that of the ME level.

When the target is not particularly low, the MCER of each firm becomes significant and thus firms have an incentive to evade taxes. Some ME activities are necessary to ensure perfect tax compliance, but in such cases, the regulator should maintain ME at a level where some firms will commit 100% tax evasion while others will not. It is unreasonable to set the ME level so that no firms will evade paying taxes entirely, as such a policy is tremendously expensive.

Some trade-offs between the aggregate abatement cost and ME cost are required at the optimum. The regulator should seek an efficient allocation of emission reductions only within the coalition of relatively compliant firms and leave the least compliant firms to earn risk rents. In order to save on ME costs, compliant firms are, in effect, forced to make larger emission reductions than noncompliant ones.

If the target reduction becomes more ambitious, the regulator needs to raise the tax rate as well as the ME level, although greater stress should be placed on use of the former than the latter. The regulator should make more of a compromise on the abatement cost efficiency in order to offset the rise in ME cost. Consequently, the proportion of firms whose MCER are equalized falls, i.e., the size of the efficient coalition becomes smaller. The following Proposition 8 treats the extreme cases where $\bar{R} \geq \bar{R}^{\#}$:

*11 We employ the term 'upper' because $\underline{a}/k$ is the lower critical level.

Proposition 8　Suppose $\bar{R} \geq \bar{R}^{\#}$, then:

(i) every environmental policy (t^*, s^*) that satisfies

$$t^* \geq k\bar{R} + \frac{\bar{a} - \underline{a}}{2},$$
$$s^* = k\bar{R} - \frac{\bar{a} + \underline{a}}{2}$$

is optimal.

(ii) the MCER of each firm j is equal to its SEEMP and smaller than t^* for all $j \in [0, 1] \times [0, 1]$ when one of the optimal policies is implemented.

When the target is very high, the MCER of each firm is correspondingly huge and every firm has enough incentive to cheat. Thus, the ME cost to ensure tax compliance rises spectacularly. In such cases, the regulator should keep the ME level as low as possible while attaining the target. In effect, all the firms will commit 100% tax evasion.*12 There are no opportunities to trade off ME cost against the aggregate abatement cost, because the marginal cost of ME is sufficiently high. The regulator should abandon the abatement cost efficiency and allow all the firms to seek risk rents. There is no efficient coalition whose size is significant. As a result, individual reductions will vary widely among firms whose technologies are the same: compliant firms are forced larger reductions than noncompliant ones. Although the idea is similar to what was discussed in the intermediate target case, it should be carried out thoroughly when the targets are extremely high.

4 Concluding Remarks

We have studied the issue of how the regulator should deal with the trade-off between the aggregate abatement cost and the ME cost when attempting to attain a predetermined target of aggregate emission reduction by way of taxation.

When the regulator implements environmental taxation, regulated polluters who maximize self-interest do not always comply with it. Each polluter will evade the tax when it considers noncompliance to be more beneficial. If some polluters commit 100% tax evasion, then the unit tax system no longer minimizes the aggregate abatement cost among all the polluters. It is possible for the regulator to strengthen ME activities to deter tax evasion and recover the abatement cost efficiency, however, the regulator must bear some extra costs in carrying out the ME.

Therefore, the regulator is faced with the choice of implementing costly ME to preserve abatement cost efficiency for the overall economy or accepting losses in the abatement costs in order to realize ME cost savings.

We have found that the optimal trade-off between the two costs crucially depends on the target levels the regulator seeks. If the target is sufficiently low to ensure tax compliance of all the polluters without costly ME, the regulator should maintain the abatement cost efficiency among all the polluters. There is no trade-off between the two costs.

If the target is relatively higher and thus some ME costs are inevitable in order to compel all

*12 This result is consistent with that of Macho-Stadler and Pérez-Castrillo (2006) where the abatement cost efficiency is not considered.

the polluters to comply, then it is too costly to raise the ME level until the abatement cost efficiency among all the polluters is ensured. Instead, the regulator should leave the least compliant polluters to commit 100% tax evasion and minimize the sum of the abatement costs within the relatively compliant polluters. Hence, some trade-offs between the two costs are required.

The higher the target becomes, the more compromise on the aggregate abatement cost the regulator needs to make. In other words, the regulator has to accept more costs generated by misallocation of emission reduction among polluters in order to moderate the rise in the ME cost. If the target is extremely high, the regulator will run out of opportunities to trade off the aggregate abatement cost against the ME cost. In this case, all the regulator can do is abandon the idea of abatement cost efficiency and attain the target.

The results of our study provide some important information for environmental policy makers who intend to adopt the 'pricing and standards' approach, that is, to realize a given target level of aggregate emission reduction via uniform unit taxation on emissions. As Shiota (2008) demonstrated, without any additional ME activities, the pricing and standards approach fails to minimize the aggregate abatement cost if the target is high. Thus, the policy maker will have to choose whether or not to strengthen the ME activities to recover the abatement cost efficiency.

Our analysis implies that, except for the cases where target reductions are exceedingly large, environmental taxation still has an advantage over non-market instruments such as fixed quotas, even if we consider the possible failure of the overall abatement cost efficiency caused by incomplete tax compliance. The unit tax system reinforced with moderate ME activities ensures efficient allocation of emission reductions among relatively compliant firms and thus will approximate the optimal trade-off between the aggregate abatement cost and the ME cost. However, when the target is extremely high, the advantage of taxation becomes ambiguous because the policymaker should completely abandon the abatement cost efficiency.

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Appendix

The appendix contains the proofs of the propositions in the text.

Proofs of Proposition 1, Proposition 2, and Proposition 3.

Given $p > 0$, $t \geq 0$, and $s \geq 0$. Let μ_j denote the Lagrange multiplier on the constraint $b_j \leq 1$. Then, the solution (x_j, e_j, b_j) must satisfy the following first-order conditions:

$$p \leq \frac{dC_j^X(x_j)}{dx_j}, \text{ with equality if } x_j > 0, \tag{20}$$

$$-k(e_j - e^0) \leq tb_j + \gamma(1 - b_j), \text{ with equality if } e_j > 0, \tag{21}$$

$$(\gamma - t)e_j \leq \mu_j, \text{ with equality if } b_j > 0, \tag{22}$$

$$\mu_j \geq 0, \text{ with equality if } b_j < 1, \tag{23}$$

$$b_j \leq 1, \text{ with equality if } \mu_j > 0. \tag{24}$$

In what follows, we complete the proofs one by one.

Proof of Proposition 1.

(2) implies that $x_j = 0$ contradicts (20). (1), (2), and (3) assure that there is a $x_j > 0$ that satisfies

$$p = \frac{dC_j^X(x_j)}{dx_j}.$$

Since

$$\frac{dx}{dp} = \frac{d^2C_j^X}{dx_j^2} > 0,$$

we understand that x_j is increasing in p. ■

Proof of Proposition 2.

Suppose that $t < \gamma$ and $e_j > 0$.

If $b_j = 0$, from (22) and (23), we have

$$(\gamma - t)e_j \leq 0.$$

Since $e_j > 0$, the inequality implies that $t \geq \gamma$. This contradicts the assumption that $t < \gamma$.

If $b_j \in (0, 1)$, from (22) and (23), we have

$$(\gamma - t)e_j = 0.$$

Since $e_j > 0$, the equation implies that $t = \gamma$. This contradicts the assumption that $t < \gamma$.

Therefore we conclude that $b_j = 1$.

Suppose that $t = \gamma$ and $e_j > 0$. From (22) and (23), we have

$$\mu_j \geq 0, \quad \text{with equality if } b_j \in (0, 1).$$

(21) becomes

$$e_j = e^0 - \frac{t}{k}.$$

There is no additional restriction as to the value of b_j: thus b_j could take any value in [0, 1]. Consequently, we have

$$e_j = e^0 - \frac{t}{k} = e^0 - \frac{\gamma}{k},$$
$$b_j \in [0, 1].$$

Suppose that $t > \gamma$ and $e_j > 0$.

If $b_j = 1$, from (22) and (23), we have

$$(\gamma - t)e_j \geq 0.$$

Since $e_j > 0$, the inequality implies that $t \leq \gamma$. This contradicts the assumption that $t > \gamma$.

If $b_j \in (0, 1)$, from (22) and (23), we have

$$(\gamma - t)e_j = 0.$$

Since $e_j > 0$, the equation implies that $t = \gamma$. This contradicts the assumption that $t > \gamma$.

Therefore we conclude that $b_j = 0$. ■

Proof of Proposition 3.

Suppose that $t < \gamma$ and $e_j > 0$. Proposition 2 indicates that $b_j = 1$. Thus, from (21), we have

$$e_j = e^0 - \frac{t}{k}.$$

Suppose that $t = \gamma$ and $e_j > 0$. Then (21) implies that

$$e_j = e^0 - \frac{t}{k} = e^0 - \frac{\gamma}{k}.$$

Suppose that $t > \gamma$ and $e_j > 0$. Proposition 2 indicates that $b_j = 0$. Thus, from (21), we have

$$e_j = e^0 - \frac{\gamma}{k}. \blacksquare$$

Proof of Proposition 4.

We have assumed that γ is distributed on the interval $[\underline{a}+s,\ \bar{a}+s]$. Thus, if $t \in [0,\ \underline{a}+s]$, then $t \leq \gamma$ for all $\gamma \in [\underline{a}+s,\ \bar{a}+s]$; if $t \in (\underline{a}+s,\ \bar{a}+s]$, then $t \leq \gamma$ for all $\gamma \in [t,\ \bar{a}+s]$ and $t > \gamma$ for all $\gamma \in [\underline{a}+s,\ t)$; and if $t > \bar{a}+s$, then $t > \gamma$ for all $\gamma \in [\underline{a}+s,\ \bar{a}+s]$.

Therefore, by Proposition 3, we know that if $t \in [0,\ \underline{a}+s]$, then $e_j(t, s) = e^0 - \frac{t}{k}$ for all $\gamma \in [\underline{a}+s,\ \bar{a}+s]$; if $t \in (\underline{a}+s,\ \bar{a}+s]$, then $e_j(t, s) = e^0 - \frac{t}{k}$ for all $\gamma \in [t,\ \bar{a}+s]$ and $e_j(t, s) = e^0 - \frac{\gamma}{k}$ for all $\gamma \in [\underline{a}+s,\ t)$; if $t > \bar{a}+s$, then $e_j(t, s) = e^0 - \frac{\gamma}{k}$ for all $\gamma \in [\underline{a}+s,\ \bar{a}+s]$.

Hence, the aggregate emission reduction becomes as follows: If $0 \leq t \leq \underline{a}+s$,

$$\begin{aligned}&\int_0^1\int_0^1 [e_j(0,0) - e_j(t,s)]\, dj_1\, dj_2 \\ &= \int_{\underline{a}+s}^{\bar{a}+s}\int_{\underline{e}^0}^{\bar{e}^0} \frac{t}{k}\frac{1}{\bar{e}^0 - \underline{e}^0}\, de^0\, \frac{1}{\bar{a}-\underline{a}}\, d\gamma = \frac{t}{k}.\end{aligned}$$

If $\underline{a}+s < t \leq \bar{a}+s$,

$$\begin{aligned}&\int_0^1\int_0^1 [e_j(0,0) - e_j(t,s)]\, dj_1\, dj_2 \\ &= \int_{\underline{a}+s}^{t}\int_{\underline{e}^0}^{\bar{e}^0} \frac{\gamma}{k}\frac{1}{\bar{e}^0 - \underline{e}^0}\, de^0\, \frac{1}{\bar{a}-\underline{a}}\, d\gamma + \int_{t}^{\bar{a}+s}\int_{\underline{e}^0}^{\bar{e}^0} \frac{t}{k}\frac{1}{\bar{e}^0 - \underline{e}^0}\, de^0\, \frac{1}{\bar{a}-\underline{a}}\, d\gamma \\ &= \frac{1}{2k(\bar{a}-\underline{a})}[2t(\bar{a}+s) - t^2 - (\underline{a}+s)^2].\end{aligned}$$

If $t > \bar{a}+s$,

$$\begin{aligned}&\int_0^1\int_0^1 [e_j(0,0) - e_j(t,s)]\, dj_1\, dj_2 \\ &= \int_{\underline{a}+s}^{\bar{a}+s}\int_{\underline{e}^0}^{\bar{e}^0} \frac{\gamma}{k}\frac{1}{\bar{e}^0 - \underline{e}^0}\, de^0\, \frac{1}{\bar{a}-\underline{a}}\, d\gamma = \frac{\bar{a}+\underline{a}+2s}{2k}. \blacksquare\end{aligned}$$

Proof of Proposition 5.

We set C_j^E as in (4). By the same token as the proof of Proposition 4, the aggregate abatement cost becomes as follows: If $0 \leq t \leq \underline{a}+s$,

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2$$
$$= \int_{\underline{a}+s}^{\bar{a}+s} \int_{\underline{e}^0}^{\bar{e}^0} \frac{t^2}{2k} \frac{1}{\bar{e}^0 - \underline{e}^0} de^0 \frac{1}{\bar{a} - \underline{a}} d\gamma = \frac{t^2}{2k}.$$

If $\underline{a} + s < t \leq \bar{a} + s$,

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2$$
$$= \int_{\underline{a}+s}^{t} \int_{\underline{e}^0}^{\bar{e}^0} \frac{\gamma^2}{2k} \frac{1}{\bar{e}^0 - \underline{e}^0} de^0 \frac{1}{\bar{a} - \underline{a}} d\gamma + \int_{t}^{\bar{a}+s} \int_{\underline{e}^0}^{\bar{e}^0} \frac{t^2}{2k} \frac{1}{\bar{e}^0 - \underline{e}^0} de^0 \frac{1}{\bar{a} - \underline{a}} d\gamma$$
$$= \frac{1}{6k(\bar{a} - \underline{a})}[3t^2(\bar{a} + s) - 2t^3 - (\underline{a} + s)^3].$$

If $t > \bar{a} + s$,

$$\int_0^1 \int_0^1 [C_j^E(e_j(t,s)) - C_j^E(e_j(0,0))]\, dj_1\, dj_2$$
$$= \int_{\underline{a}+s}^{\bar{a}+s} \int_{\underline{e}^0}^{\bar{e}^0} \frac{\gamma^2}{2k} \frac{1}{\bar{e}^0 - \underline{e}^0} de^0 \frac{1}{\bar{a} - \underline{a}} d\gamma$$
$$= \frac{\bar{a}^2 + \bar{a}\underline{a} + \underline{a}^2 + 3s(\bar{a} + \underline{a}) + 3s^2}{6k}. \blacksquare$$

Proof of Lemma 1.

Let s'' be

$$s'' = \max\{t' - \underline{a}, 0\}. \tag{25}$$

Then we have $s'' < s'$. Because (13) implies that $F(s'') < F(s')$, we know that the ME cost of the policy (t', s'') is strictly lower than that of (t', s').

(25) also implies that $t' \leq \underline{a} + s''$. Then, from Proposition 4, we know that not only the policy (t', s') but also (t', s'') reduces as much emissions as $\frac{t'}{k}$.

In addition, from Proposition 5, we know that the aggregate abatement cost of (t', s'') amounts to $\frac{t'^2}{2k}$, which is exactly the same as that of (t', s').

Consequently, there is a policy (t', s'') which realizes the same amount of emission reductions as (t', s'), and whose total cost is strictly lower than that of (t', s'). Therefore, we conclude that (t', s') is not optimal. $\blacksquare$

Proof of Lemma 2.

Proposition 4 implies that the policy (t^*, s^*) as well as (t^{**}, s^*) reduces as much emissions as $\frac{\bar{a}+\underline{a}+2s^*}{2k}$, since both t^* and t^{**} are not smaller than $\bar{a} + s^*$.

Additionally, Proposition 5 shows that the aggregate abatement cost of (t^*, s^*) amounts to $\frac{1}{6k}[\bar{a}^2 + \bar{a}\underline{a} + \underline{a}^2 + 3s^*(\bar{a} + \underline{a}) + 3(s^*)^2]$, which is the same as that of (t^{**}, s^*), since both t^* and t^{**} are not smaller than $\bar{a} + s^*$. ■

Proof of Proposition 6.

The policy $(t, s) = (k\bar{R}, 0)$ satisfies $t \le \underline{a} + s$ since $k\bar{R} \le \underline{a} + 0$ from $\bar{R} \le \underline{a}/k$. Thus, from Proposition 4, it is clear that $(k\bar{R}, 0)$ realizes the target $\bar{R}$.

When $(k\bar{R}, 0)$ is implemented, γ is distributed on the interval $[\underline{a},\ \bar{a}]$. Since $t = k\bar{R}$ and $\bar{R} \le \underline{a}/k$, $\gamma \ge t$ for all the firms. Then, Proposition 3 shows that the MCER of each firm is equal to t for all the firms. Since each firm j is represented by the point $j \in [0, 1] \times [0, 1]$, we can restate the point of the last sentence as follows: the MCER of each firm j is equal to the tax rate for all $j \in [0, 1] \times [0, 1]$. Therefore, the aggregate abatement cost of $(k\bar{R}, 0)$ is minimized among the policies that attain $\bar{R}$.

Further, (13) indicates that the ME cost of $(k\bar{R}, 0)$ is also minimized.

Consequently, the total cost of $(k\bar{R}, 0)$ is minimized among the policies that attain $\bar{R}$. Hence, we conclude that $(k\bar{R}, 0)$ is optimal. ■

Proof of Proposition 7 - (i) and Proposition 8 - (i).

Multiplying the objective function of RRP by -1, we transform RRP into the equivalent maximization problem (EMP).

Let ν, ϕ, and ψ denote the Lagrange multipliers associated with the constraints (14), (16), and (17), respectively. We can derive the following first-order conditions of EMP:

$$\frac{1}{k(\bar{a}-\underline{a})}[t^2 - t(\bar{a} + s)] + \frac{\nu}{k(\bar{a}-\underline{a})}(\bar{a} + s - t) + \phi - \psi = 0, \tag{26}$$

$$\frac{1}{2k(\bar{a}-\underline{a})}[(\underline{a} + s)^2 - t^2] - F'(s) + \frac{\nu}{k(\bar{a}-\underline{a})}(t - \underline{a} - s) - \phi + \psi \le 0,$$
$$\text{with equality if } s > 0, \tag{27}$$

$$\nu \ge 0, \quad \text{with equality if (14) holds with strict inequality,} \tag{28}$$

$$\phi \ge 0, \quad \text{with equality if (16) holds with strict inequality,} \tag{29}$$

$$\psi \ge 0, \quad \text{with equality if (17) holds with strict inequality.} \tag{30}$$

We examine the following six possible cases, respectively:

$$
\begin{array}{ll}
\text{(i)} & t \in (\underline{a} + s,\ \bar{a} + s) \text{ and } s > 0 \\
\text{(ii)} & t = \underline{a} + s \text{ and } s > 0 \\
\text{(iii)} & t = \bar{a} + s \text{ and } s > 0 \\
\text{(iv)} & t \in (\underline{a},\ \bar{a}) \text{ and } s = 0 \\
\text{(v)} & t = \underline{a} \text{ and } s = 0 \\
\text{(vi)} & t = \bar{a} \text{ and } s = 0
\end{array}
$$

We will find that only case (i) and (iii) have possible maximizers of EMP.

Case (i).

(29) and (30) show $\phi = \psi = 0$. Then, from (26), we have $t = v$. Considering these in (27), we have

$$(t - \underline{a} - s)^2 = 2k(\bar{a} - \underline{a})F'(s).$$

Since $t > \underline{a} + s$,

$$t = \underline{a} + s + \sqrt{2k(\bar{a} - \underline{a})F'(s)}. \tag{31}$$

$\underline{a} > 0$ implies $t > 0$ and $v > 0$. Thus (14) holds with equality. Putting (31) into (14), we have the value of s.

Considering $t < \bar{a} + s$ in (31), we have

$$\underline{a} + s + \sqrt{2k(\bar{a} - \underline{a})F'(s)} < \bar{a} + s,$$

which reduces to

$$2kF'(s) < \bar{a} - \underline{a}. \tag{32}$$

Letting $\tilde{s}$ be the number given by $2kF'(\tilde{s}) = \bar{a} - \underline{a}$, (32) means $s < \tilde{s}$. Then, we have

$$\underline{a} + \tilde{s} + \sqrt{2k(\bar{a} - \underline{a})F'(\tilde{s})} = \bar{a} + \tilde{s}.$$

It is clear $t < \bar{a} + \tilde{s}$ holds if (31) and $s < \tilde{s}$ hold.

Because of Proposition 4 and Definition 2, we know $\bar{R} = \bar{R}^{\#}$ when $(t, s) = (\bar{a} + \tilde{s},\ \tilde{s})$. Therefore, $\bar{R} < \bar{R}^{\#}$ is necessary so that (31) and $t < \bar{a} + s$ will hold.

Because $t \in (\underline{a} + s,\ \bar{a} + s)$, the binding constraint is (14) alone in Case (i). Considering $v = t$, we can write the bordered Hessian $BH_{(\mathrm{i})}$ as follows:

$$BH_{(\mathrm{i})} = \begin{pmatrix} \frac{t-\bar{a}-s}{k(\bar{a}-\underline{a})} & 0 & \frac{\bar{a}+s-t}{k(\bar{a}-\underline{a})} \\ 0 & \frac{\underline{a}+s-t}{k(\bar{a}-\underline{a})} - F''(s) & \frac{t-\underline{a}-s}{k(\bar{a}-\underline{a})} \\ \frac{\bar{a}+s-t}{k(\bar{a}-\underline{a})} & \frac{t-\underline{a}-s}{k(\bar{a}-\underline{a})} & 0 \end{pmatrix}. \tag{33}$$

Since $t \in (\underline{a} + s,\ \bar{a} + s)$ and $F''(s) > 0$, the sign of each element is

$$\begin{pmatrix} - & 0 & + \\ 0 & - & + \\ + & + & 0 \end{pmatrix}$$

Thus, we have

$$|BH_{(\mathrm{i})}| > 0, \tag{34}$$

where $|BH_{(\mathrm{i})}|$ denotes the determinant. This shows that the solution in case (i) is a strict local maximizer of EMP.

Therefore, case (i) has a candidate of the solution of EMP if and only if $\bar{R} < \bar{R}^{\#}$.

Case (ii).

From $t = \underline{a} + s$ and (14) with equality, we have $s = k\bar{R} - \underline{a}$ and $t = k\bar{R}$.

Since $t = \underline{a} + s < \bar{a} + s$, (30) shows $\psi = 0$.

Considering t, s, and ϕ in (27), we have

$$\phi = -F'(k\bar{R} - \underline{a}) < 0,$$

because we suppose $s = k\bar{R} - \underline{a} > 0$. However, $\phi < 0$ contradicts (29).

Hence, there is no solution for EMP in case (ii).

Case (iii).

From $t = \bar{a} + s$ and (14), we have

$$t = k\bar{R} + \frac{\bar{a} - \underline{a}}{2}, \tag{35}$$

$$s = k\bar{R} - \frac{\bar{a} + \underline{a}}{2}. \tag{36}$$

Since $t = \bar{a} + s > \underline{a} + s$, (29) shows $\phi = 0$.

Considering t, s, and ϕ in (26) and (27), we have

$$v = k[F'(k\bar{R} - \frac{\bar{a} + \underline{a}}{2}) + \bar{R}]. \tag{37}$$

Let $(\hat{t}, \hat{s}, \hat{v})$ be a solution of (35), (36), and (37). We now consider a variant problem VP which maximizes the same objective function as EMP, subject only to (14). It is clear that $(\hat{t}, \hat{s}, \hat{v})$ satisfies the first order conditions of VP. The bordered Hessian of VP, evaluated at $(\hat{t}, \hat{s}, \hat{v})$ is as follows:

$$\begin{pmatrix} \frac{\bar{a}+\hat{s}-\hat{v}}{k(\bar{a}-\underline{a})} & \frac{\hat{v}-\bar{a}-\hat{s}}{k(\bar{a}-\underline{a})} & 0 \\ \frac{\hat{v}-\bar{a}-\hat{s}}{k(\bar{a}-\underline{a})} & \frac{\underline{a}+\hat{s}-\hat{v}}{k(\bar{a}-\underline{a})} - F''(\hat{s}) & \frac{1}{k} \\ 0 & \frac{1}{k} & 0 \end{pmatrix}. \tag{38}$$

Its determinant is

$$\frac{\hat{v}-\bar{a}-\hat{s}}{k^3(\bar{a}-\underline{a})} = \frac{1}{k^3(\bar{a}-\underline{a})}\left[kF'\left(k\bar{R} - \frac{\bar{a}+\underline{a}}{2}\right) - \frac{\bar{a}-\underline{a}}{2}\right], \tag{39}$$

whose sign depends on that of

$$F'(k\bar{R} - \frac{\bar{a}+\underline{a}}{2}) - \frac{\bar{a}-\underline{a}}{2k}.$$

From Definition 2, (18), and (19), we understand that the sign of (39) depends on the target level $\bar{R}$.

Suppose $\bar{R} < \bar{R}^{\#}$. Then the sign of (39) is negative and thus $(\hat{t}, \hat{s})$ gives a strict local minimum of VP. Because $(\hat{t}, \hat{s})$ also satisfies the constraints of EMP, it gives a strict local minimum of EMP as well. Then there is a point around $(\hat{t}, \hat{s})$ that gives a strictly larger value to the objective function of EMP. Hence, $(\hat{t}, \hat{s})$ will not be a maximizer of EMP when $\bar{R} < \bar{R}^{\#}$.

Suppose $\bar{R} > \bar{R}^{\#}$. Then the sign of (39) is positive and thus $(\hat{t}, \hat{s})$ gives a strict local maximum of VP. Because $(\hat{t}, \hat{s})$ also satisfies the constraints of EMP, it gives a strict local maximum of EMP as well.

Suppose $\bar{R} = \bar{R}^{\#}$. Then the value of (39) is zero. After performing permutations on (38), we get another determinant which is exactly the same as (39). Thus, the necessary condition for $(\hat{t}, \hat{s})$ to be a local maximizer of VP holds.

Therefore, case (iii) has a candidate of the solution of EMP if and only if $\bar{R} \geq \bar{R}^{\#}$.

Case (iv).

(29) and (30) show $\phi = \psi = 0$. Then, from (26), we have $t = v$.

Considering ϕ, ψ, and v in (27), we have

$$(t - \underline{a})^2 \leq 2k(\bar{a} - \underline{a})F'(0).$$

Because of (13), this inequality becomes

$$(t - \underline{a})^2 \leq 0.$$

However, it contradicts our assumption $t > \underline{a}$.

Hence, there is no solution for EMP in case (iv).

Case (v).

Proposition 4 shows that the policy $(\underline{a}, 0)$ can reduce emissions in the aggregate no more than $\underline{a}/k$. It is not compatible with the target of this problem since we have assumed $\bar{R} > \underline{a}/k$.

Hence, there is no solution for EMP in case (v).

Case (vi).

(29) implies $\phi = 0$.

Because $t = \bar{a}$ and $s = 0$, (14) shows that Case (vi) holds only if

$$\bar{R} = \frac{\bar{a} + \underline{a}}{2k}. \tag{40}$$

We suppose that (40) holds in the following.

Considering ϕ, t, s in (26), we get $\psi = 0$.

Then, (27) and (28) shows

$$v \in \left[0, \ \frac{\bar{a} + \underline{a}}{2}\right] \tag{41}$$

We now consider a variant problem VP which maximizes the same objective function as EMP such that (14) and (40). It is clear that $(t, s, v) = (\bar{a}, 0, (\bar{a} + \underline{a})/2)$, a solution in case (vi), satisfies the first order conditions of VP. The bordered Hessian of VP, evaluated at $(\bar{a}, 0, (\bar{a} + \underline{a})/2)$ is as follows:

$$\begin{pmatrix} \frac{1}{2k} & -\frac{1}{2k} & 0 \\ -\frac{1}{2k} & -\frac{1}{2k} - F''(0) & \frac{1}{k} \\ 0 & \frac{1}{k} & 0 \end{pmatrix}. \tag{42}$$

Because the determinant of (42) is

$$-\frac{1}{2k^3} < 0, \tag{43}$$

$(\bar{a}, 0, (\bar{a}+\underline{a})/2)$ gives a strict local minimum of VP. Because $(\bar{a},\ 0)$ also satisfies the constraints of EMP, it gives a strict local minimum of EMP as well. Then there is a point around $(\bar{a},\ 0)$ that gives a strictly larger value to the objective function of EMP. Therefore, $(\bar{a},\ 0)$ will not be a maximizer of EMP.

We do not have to examine the case where $v \in \left[0,\ \frac{\bar{a}+\underline{a}}{2}\right)$, because v does not affect the value of the objective function in EMP.

Hence, there is no solution of EMP in case (vi).

From the above, we understand that if $\bar{R} < \bar{R}^{\#}$, then case (i) has a candidate of a maximizer of EMP and the rest of the cases do not have any candidate, and that if $\bar{R} \geq \bar{R}^{\#}$, then case (iii) has a candidate and the rest of the cases do not.

Considering Lemma 2 additionally, we can conclude that if $\bar{R} < \bar{R}^{\#}$, then the optimal environmental policy (t^*, s^*) satisfies $t^* \in (\underline{a} + s^*,\ \bar{a} + s^*)$ and $s^* > 0$, and that if $\bar{R} \geq \bar{R}^{\#}$, then (t^*, s^*) satisfies $t^* \geq \bar{a} + s^*$ and $s^* > 0$. ■

Proof of 7 - (ii).

When $(t^*,\ s^*)$ is implemented, γ is distributed on the interval $[\underline{a}+s^*,\ \bar{a}+s^*]$. Since $t^* \in (\underline{a}+s^*,\ \bar{a}+s^*)$, $\gamma \geq t^*$ for all the firms with $\gamma \in [t^*,\ \bar{a}+s^*]$, and $\gamma \leq t^*$ for all the firms with $\gamma \in [\underline{a}+s^*,\ t^*]$.

Then, Proposition 3 reveals that the MCER is equal to t^* for all the firms with $\gamma \in [t^*,\ \bar{a}+s^*]$, and is equal to the SEEMP and smaller than t^* for all the firms with $\gamma \in [\underline{a}+s^*,\ t^*]$.

From Assumption 2, we have $j_2 = (\gamma - \underline{a} - s^*)/(\bar{a} - \underline{a})$. Hence, we can restate the point as follows: the MCER of each firm j is equal to t^* for all $j \in [0,1] \times [(t^* - \underline{a} - s^*)/(\bar{a} - \underline{a}),\ 1]$, and is equal to its SEEMP and smaller than t^* for all $j \in [0,1] \times [0,\ (t^* - \underline{a} - s^*)/(\bar{a} - \underline{a})]$. ■

Proof of Proposition 7 - (iii).

We study the comparative statics effects of $\bar{R}$ on t and s respectively, and then, compare their magnitude.

From (26) - (30), and Proposition 7 - (i), we know that, at the optimum, (t, s, v) satisfies the following equations:

$$\frac{1}{k(\bar{a}-\underline{a})}[t^2 - t(\bar{a}+s)] + \frac{v}{k(\bar{a}-\underline{a})}(\bar{a}+s-t) = 0,$$

$$\frac{1}{2k(\bar{a}-\underline{a})}[(\underline{a}+s)^2 - t^2] - F'(s) + \frac{v}{k(\bar{a}-\underline{a})}(t-\underline{a}-s) = 0,$$

$$\frac{1}{2k(\bar{a}-\underline{a})}[2t(\bar{a}+s) - t^2 - (\underline{a}+s)^2] = \bar{R}.$$

Totally differentiating these expressions and considering $v = t$, we yield the following:

$$BH_{(i)} \begin{pmatrix} dt \\ ds \\ dv \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 1 \end{pmatrix} d\bar{R}, \tag{44}$$

where $BH_{(i)}$ is defined in (33). From (34), we know $|BH_{(i)}|$, the determinant, is positive.

Because $t \in (\underline{a}, \bar{a})$, and $F''(s) > 0$, we get the following results:

$$\frac{\partial t}{\partial \bar{R}} = \frac{1}{|BH_{(i)}|}\left[\frac{\bar{a}+s-t}{k(\bar{a}-\underline{a})}\right]\left[F''(s) + \frac{t-\underline{a}-s}{k(\bar{a}-\underline{a})}\right] > 0,$$

which implies that the regulator should raise the tax rate to attain a higher target.

$$\frac{\partial s}{\partial \bar{R}} = \frac{1}{|BH_{(i)}|}\left[\frac{\bar{a}+s-t}{k(\bar{a}-\underline{a})}\right]\left[\frac{t-\underline{a}-s}{k(\bar{a}-\underline{a})}\right] > 0,$$

which implies that the regulator should raise ME level to attain a higher target.

$$\frac{\partial t}{\partial \bar{R}} - \frac{\partial s}{\partial \bar{R}} = \frac{1}{|BH_{(i)}|}\left[\frac{\bar{a}+s-t}{k(\bar{a}-\underline{a})}F''(s)\right] > 0,$$

which implies that the rise in the tax rate should exceed that in the ME level. ■

Proof of 8 - (ii).

Suppose (t^*, s^*) satisfies

$$t^* \geq k\bar{R} + (\bar{a}-\underline{a})/2,\ s^* = \bar{R} - (\bar{a}+\underline{a})/2.$$

When (t^*, s^*) is implemented, γ is distributed on the interval $[k\bar{R} - (\bar{a}-\underline{a})/2,\ k\bar{R} + (\bar{a}-\underline{a})/2]$. Since $t^* \geq k\bar{R} + (\bar{a}-\underline{a})/2$, $\gamma \leq t^*$ for all the firms.

Then, Proposition 3 reveals that the MCER of each firm is equal to its SEEMP and smaller than the tax rate for all the firms.

Since each firm j is represented by the point $j \in [0, 1] \times [0, 1]$, we can restate that the MCER of each firm j is equal to its SEEMP and smaller than t^* for all $j \in [0, 1] \times [0, 1]$. ■

総汚染削減費用と監視・強制費用との最適トレードオフ

塩田　尚樹

本研究では，所与の総汚染削減目標を達成する際に総汚染削減費用と監視・強制費用との和を最小化しようとする政府機関の行動をモデル化し，目標総汚染削減量と最適な環境税率および監視・強制活動の水準との関係について検討した。完全な脱税者が発生すると環境税の利点である総汚染削減費用の最小化が達成されないことがShiota(2008)で証明されているが，監視・強制費用を抑えるために完全な脱税者の発生をどの程度許容するかが本論でのポイントとなった。分析の結果，完全な脱税者がゼロとなる水準まで監視・強制活動を強化すると，目標削減量が極端に少ない場合を除いて，社会的費用がかえって大きくなることが判った。目標達成にともなう社会的費用を抑えるためには，脱税者の発生をある程度容認して，総汚染削減費用と監視・強制費用とのバランスをとることが重要である。よって，汚染削減量の効率的な割り当ては，相対的に遵法意識の高い汚染排出企業の提携内だけで考慮するべきである。なお，総削減目標が大になるほど，総汚染削減費用の抑制よりも監視・強制費用の抑制を重視する必要がある。